International Journal of Theoretical Physics, Vol. 39, No. 12, 2000

Correlation of the Flux Tube Constant with the Nucleon Electric Polarizability

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Received April 11, 2000

A three-quark shell model of the nucleon is used to calculate analytically the polarizability using a scalar linear flux tube potential with no one-gluon exchange potential included. A value of $12.02 \times 10^{-4}\ f^3$ is obtained for the proton, in good agreement with experiment, with the flux tube constant adjusted to reproduce the proton Δ average rest energy. The magnetic polarizability of the proton is then calculated as $1.51 \times 10^{-4}\ f^3$, which is in agreement with the experimental value. The neutron/proton electric polarizability ratio is calculated as 2/3, and the neutron electric polarizability is predicted to be $8.01 \times 10^{-4}\ f^3$.

1. INTRODUCTION

An external electric field will accelerate a nucleon and also distort the shape of a distribution of charged quarks. The stronger the force between the quarks, the less a given electric field will change the shape of a quark distribution in a nucleon. The proton electric polarizability thus serves as an experimental way of obtaining information about the strong forces acting on quarks. Likewise, an external magnetic field will tend to distort a distribution of quark currents, allowing the magnetic polarizability to serve as a second way to probe the strong forces on quarks within a nucleon. The experimental values for the proton electric [15, 18] and magnetic polarizabilities are about $12.2 \times 10^{-4}\ f^3$ and $2.1 \times 10^{-4}\ f^3$, respectively. The polarizabilities for the neutron are more difficult to obtain, but there is a value [24] of $(12.0 \pm 1.5 \pm 2.) \times 10^{-4}\ f^3$ based on neutron scattering on heavy nuclei. There is a more recent value [17] of $(0, \pm 5)\ 10^{-4}\ f^3$. There is also the sum rule result [8, 14] finding the proton minus the neutron electric polarizability is

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0020-7748/00/1200-2853$18.00/0

correlated to the proton rms charge radius, resulting in the difference being $3.8 \times 10^{-4}\ f^3$. Chiral perturbation theories [5, 6, 8, 16, 22] based on the photon interacting mostly with the pion clouds surrounding the nucleon find a neutron/proton polarizability ratio of 1:1 in agreement with data on neutron scattering on heavy nuclei. Using the experimental electric polarizability for the proton with the sum rule estimates a neutron electric polarizability of about 2/3 that of the proton. Assuming the photons interact mainly with the quarks in a nucleon results in a neutron/proton electric polarizability ratio of the same 2/3 value. That motivates this paper to determine the polarizability of the proton using a three-quark model for the nucleon.

In a relativistic three-quark model, the charge and spin of the proton are modeled by assigning appropriate charges to the assumed three-quark constituents and assigning them to a $(1/2^+)^3$ configuration coupled to a total angular momentum of one half. The configuration is labeled by the upper component quantum numbers for each quark. The three-body Dirac equation is used to describe the dynamics of the bound system and to determine the composite three-quark wave function. The lower component of the Dirac wave function for a bound quark, neglected in nonrelativistic constituent quark approaches, is needed to determine the magnetic polarizability. The quark masses in the nucleon will be assumed small.

Semay and Ceuleneer [25] used a linear scalar diagonal confinement potential in the same particle–antiparticle two-body system, obtaining absolutely confined states in describing mesons as two-quark systems. A flux tube potential proportional to the minimum length of a Y-shaped tube connecting the three quarks for any given quark locations has been used in three-valence-quark studies of the nucleon [10]. This is a three-body potential as the potential energy depends on the simultaneous location of each of the three quarks. Here a linear scalar potential

$$S = b(\beta_1 r_1 + \beta_2 r_2 + \beta_3 r_3) \tag{1}$$

is used. The flux tube constant is b. Here β is the Dirac matrix and the subscripts are particle labels. A three-body Dirac shell model is used with this scalar linear flux tube potential and massless quarks to estimate the electric and magnetic polarizabilities of the nucleon. Approximate solutions are found that are simple analytic functions(Gaussians) for use in determining the nucleon polarizabilities. Similiar assumptions are used with the three-body Dirac equation to obtain the nucleon rest energy using this scalar linear flux tube potential model.

The program of perturbative calculations needed to obtain the polarizability is to solve the unperturbed Hamiltonian H_0 for the ground-state energy E_0 and for the unperturbed wave function Ψ_0. Then one introduces as a perturbation H' the interaction of the quarks with an external electric or

magnetic field, each separately assumed in turn to be in the z direction. Then one obtains the second-order correction to the eigenvalue W_2 and the polarization is defined as the coefficient of the squared electric field strength for the electric polarizability and as a coefficient of the squared magnetic field strength for the magnetic polarizability.

Following Schiff [23], one can determine the second-order correction to the eigenvalue in one of two ways. The first is to determine the excited-state wave functions and energies of all the states with overlap to the ground state through the electric or magnetic perturbation. Then the squares of the perturbation matrix element between the ground state and each excited state divided by the excitation energy, summed over the excited states, leads to the second-order perturbation to the energy. An alternative way is to solve for the first-order correction to the wave function by solving

$$[H_0 - W_0]\Psi_1 = [W_1 - H']\Psi_0 \tag{2}$$

where W_1 is the first-order correction to the eigenvalue. W_1 vanishes for the external electric field perturbation due to parity considerations. In the three-quark model used here for the nucleon, the perturbative interaction is

$$H' = \sum_i q_i r_i \cos(\theta_i) E_z \tag{3}$$

where E_z is the external electric field and q_i is the charge of the ith quark, located at $\vec{r}_i$. The sum over i denotes the contributions from each of the three quarks. The external magnetic field interaction is

$$H' = -\sum_i q_i c\vec{r}_i \otimes \vec{\alpha}_i \cdot \vec{B}_z \tag{4}$$

where α_i is the Dirac alpha matrix for the ith quark. The first-order correction to the energy W_1 is not zero for the magnetic perturbation. Once the first-order correction to the wave function has been determined, then the second-order correction to the energy can be found from

$$W_2 = \langle \Psi_1 H' \Psi_0 \rangle \tag{5}$$

This latter method is the one followed here. The three-quark Dirac equation is solved approximately here to obtain simple wave functions for the bound current quarks. A three-quark shell model is then used to describe the nucleon. This approach allows the first-order correction to the quark wave function to be determined analytically as well as the second-order correction to the energy.

2. DETERMINING THE PROTON ELECTRIC POLARIZABILITY

The electric perturbation is

$$H' = \sum_i q_i r_i \cos(\theta_i) E_z \tag{6}$$

where E_z is the external electric field. The sum is over the three quarks assumed in the nucleon. One term at a time of this perturbation is considered. Each term of the perturbation has the same contribution to W_2, scaled only by the squared quark charge, q_i^2.

With a shell model wave function for the three quarks in the $(1/2^+)^3$ configuration for the proton, the one-body Dirac Hamiltonian is

$$H_{1\text{body}} = \vec{\alpha} \cdot \vec{p} + \beta br \tag{7}$$

One would like to solve,

$$[H_{1\text{body}} + H']\Psi = E\Psi \tag{8}$$

where E is now the energy in the presence of the above perturbation and $H_{1\text{body}}$ is the one-quark Hamiltonian. Even in perturbation theory one has large coupled off-diagonal terms to deal with when solving for the first-order correction to the wave function.

The first-order correction to the wave function is calculated using the square of the one-body Hamiltonian. The idea is to follow Abe and Fujita [1] and to operate twice with the Hamiltonian on the ground-state wave function. We let $H_0 = H_{1\text{body}}^2$ and $W_0 = E_0^2$, and include the off-diagonal terms to first order only. Dropping the ith subscript from the perturbation summation, we have

$$H_0 = p^2 + b^2 r^2 + ib\beta\alpha_r \tag{9}$$

Here α and β are the Dirac matrices, and $\alpha_r = \vec{\alpha} \cdot \hat{r}$. One now solves

$$H_0\Psi_0 = W_0\Psi_0 \tag{10}$$

After integrating over angles and spin coordinates, one finds the upper and lower uncoupled eigenfunctions by considering the diagonal elements only of H_0. These eigenfunctions are normalized harmonic oscillator eigenfunctions F_u and G_u, respectively. The corresponding eigenvalues are $(L + 3/2)2b$, where L is the orbital angular momentum of a given component. We define the usual Dirac constant

$$k = \pm(J + 1/2) \tag{11}$$

The off-diagonal terms are included to first order by considering

$$\begin{bmatrix} H_k - W_0 & b \\ b & H_{k-1} - W_0 \end{bmatrix} \begin{bmatrix} F_0 \\ G_0 \end{bmatrix} = 0 \tag{12}$$

Here H_k is given by

$$H_k = -r^{-2}\, d/dr\, [r^2\, d/dr] + b^2 r^2 + k(k+1)/r^2 \tag{13}$$

The unperturbed wave functions are component coefficients times the uncoupled solution, and $F_0 = AF_u$ and $G_0 = BG_u$. The component coefficients and the eigenvalue W_0 are determined from

$$\begin{bmatrix} 3b - W_0 & b_{12} \\ b_{12} & 5b - W_0 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = 0 \tag{14}$$

where

$$b_{12} = b[8/3\pi]^{1/2} \tag{15}$$

The results are $A^2 = 0.8677$, $B^2 = 0.1323$, and

$$W_0/b = 4 - [1 + (8/3\pi)]^{1/2} = 2.6403 \tag{16}$$

compared to Critchfield's [11] exact result 2.6226. The diagonal terms of H_0, $3b$, and $5b$ are larger than the off-diagonal term b_{12}. This is the advantage of calculating with the squared Hamiltonian compared to the linear Hamiltonian. The unperturbed wave functions are F_0 and G_0, each with a simple Gaussian, or r times a Gaussian form, with known coefficients, which will be used to determine the polarizabilities.

The calculation of the first-order correction to the wave function is then done with these wave functions. Consider

$$[H_0]\Psi = [E - H']^2\Psi \tag{17}$$

With perturbation theory, the eigenvalue term becomes

$$[E - H']^2 = E_0^2 - 2E_0H' + H'^2 \tag{18}$$

and we neglect the last term. Following second-order perturbation theory [23], we now consider

$$[H_0 - W_0]\Psi_1 = -2E_0qr\cos(\theta)E_z\Psi_0 \tag{19}$$

This is solved including only the diagonal terms of H_0. It is useful to define

$$H'' = 2E_0H' \tag{20}$$

To determine the first-order correction to the wave function Ψ_1, we have to expand the inhomogenous angular part of Eq. (19) as

$$\cos(\theta)\ Y^{-}_{1/2m} = A_{1/2}Y^{+}_{1/2m} + A_{3/2}Y^{-}_{3/2m} \tag{21}$$

and

$$\cos(\theta)\ Y^{+}_{1/2m} = B_{1/2}Y^{-}_{1/2m} + B_{3/2}Y^{+}_{3/2m} \tag{22}$$

Here $Y^{\pm}_{jm}$ are the joint eigenfunctions of quark orbital and intrinsic angular momentum coupled to a total angular momentum j, z component m, with parity $\pm$ specified by the superscript ω. Parity is $(-1)^{(j+\omega/2)}$. Each term of the inhomogenous part is separately solved for in determining Ψ_1. The angular part of Ψ_1 is chosen to match the particular inhomogenous term being considered. The radial part, R_1, of this piece of the first-order corrected wave function is then found by taking the radial part as

$$R_1 = \exp(-br^2/2)\ \Sigma\ P_n r^n \tag{23}$$

Substituting this expansion into the left-hand side of Eq. (19) results [with Eq. (13)] in algebraic equations that can be solved for the polynomial coefficients P_n. For each piece of the first-order corrected wave function, only one or two polynomial coefficients are different from zero. Then the contributions are added together, and W_2 is found by

$$W_2 = \langle \Psi_1 | H'' | \Psi_0 \rangle \tag{24}$$

Since H'' is $2E_0H'$, the electric polarizability α is found from

$$-(1/2)\alpha\ E_z^2 2E_0 = W_2 \tag{25}$$

The electric polarizability for the proton is shown in Fig. 1 versus the flux tube constant b. The neutron polarizability is 2/3 the proton polarizability, since each is proportional to the sum of the squared quark charges contained in each nucleon.

3. DETERMINING THE PROTON MAGNETIC POLARIZABILITY

The magnetic polarizability calculation proceeds along the same lines as in the above section except that the first-order correction to the energy W_1 is not zero. The interaction for each quark is

$$H' = -\mu \cdot \vec{B} = -qc\vec{r} \otimes \vec{\alpha} \cdot \vec{B} \tag{26}$$

Assuming the magnetic field B_z is in the z direction only, we obtain for the first-order correction to the energy

$$W_1 = qcB_z(2/3)AB \int_0^\infty F_u r G_u r^2\, dr \tag{27}$$

Now, in Eq. (2) we expand the inhomogenous terms of $[W_1 2E_0 - 2E_0H']\Psi_0$

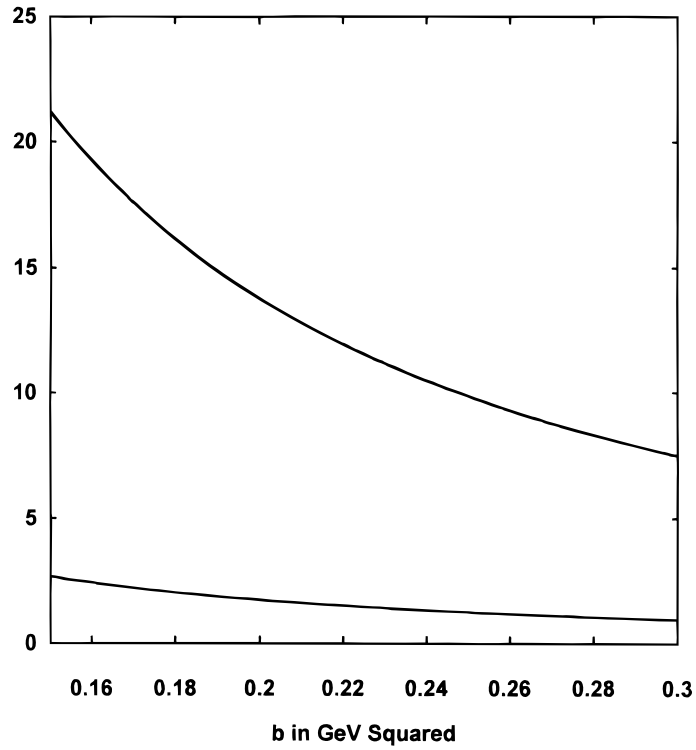

Fig. 1. Proton electric polarizability versus flux tube constant with massless quarks (upper curve). The units are $10^{-4}\, f^3$. The proton magnetic polarizability is the lower curve.

as coefficients times each of numerous j, parity inhomogenous term angular, spin eigenfunctions $Y^{\pm}_{jm}$. The contribution of each inhomogenous term to Ψ_1 is then determined, neglecting the off-diagonal b terms to H_0, as in the prior section. With

$$H'' = -2E_0 \sum_i q_i \vec{cr}_i \otimes \vec{\alpha}_i \cdot \vec{B} \tag{28}$$

one then finds

$$W_2 = \langle \Psi_1 | H'' | \Psi_0 \rangle \tag{29}$$

The magnetic polarizability β is then found from

$$-(1/2)\beta B_z^2 2E_0 = W_2 \tag{30}$$

The magnetic polarizability can be seen in Fig. 1 as the lower curve, in units of $10^{-4}\, f^3$.

4. THEORY FOR THE THREE-QUARK WAVE FUNCTION OF THE NUCLEON

The three-body Dirac equation is solved in hypercentral approximation, using hyperspherical coordinates. The six space coordinates necessary to specify the location of the particles are taken as a hyperradius ρ and five hyperangles Ω. The root mean square hyperradius value is somewhat akin to the classical concept of the radius of gyration. The hyperradius is defined as

$$\rho^2 = (r_{12}^2 + r_{23}^2 + r_{31}^2)/3 = r_1^2 + r_2^2 + r_3^2 - 3R^2 \tag{31}$$

where r_1, r_2, and r_3 are the locations of the three particles, respectively, and R is the location of the center of mass. r_{12} is the separation of particles 1 and 2, etc. The details of the hyperspherical approach can be found in ref. 19. This method as applied to the many-body Schrödinger equation has been summarized by Baz and Zhukov [4] and Ripelle [21, 3]. The hypercentral approximation utilizes the hyperangular average of the $\Sigma_{i<j}\ V_{ij}$ potential terms. The hyperangular reduction of these equations has been reported elsewhere [26–29].

The three-fermion composite wave function Ψ is written as

$$\Psi = \Sigma\ U(\Omega)R(\rho) \tag{32}$$

where the sum is over the various configurations. In general, the sum over configurations is eventually truncated by a multibody angular momentum barrier that favors small orbital angular momentum configurations for short-ranged forces. The hypercentral approximation truncates this sum to that of a single configuration, the $(1/2^+)^3$. $U(\Omega)$ is a product of the orbital, spin, flavor, and color parts of the wave function for each of the particles, and includes the angular momentum coupling. Ω denotes the hyperangles and the other spin, flavor, and color coordinates of the system. The angular momentum coupling is $[j_1, j_2]J_{12}, j_3 JM_z\rangle$, where j_1, j_2, and j_3 are the total angular momenta of each of the three particles, and J_{12} is the intermediate coupling of the first pair. The total angular momentum of the third particle is coupled to J_{12} to produce J, the total angular momentum of the three-body system, and its z component M_z. Sums over the m values are understood. For the nucleon, J is one half, and J_{12} can be only zero or one for the configurations considered here. Doing the hyperangular integration results in the three-body Dirac equation becoming a set of coupled differential equations involving derivatives with respect to the hyperradius. An eight by eight matrix is obtained for the Hamiltonian which operates on the composite three-body wave function involving products of the upper and lower components for each single-particle wave function. The unknown hyperradial dependence is symmetric upon exchange of any pair of coordinates.

The factors that make up the $U(\Omega)$ part of the composite wave function are now expressed in detail. The color singlet part of the composite wave function of three quarks can be written as a factor

$$\psi_{\text{color}} = \det(abc)/\sqrt{6} \tag{33}$$

where a, b, and c denote the three color indices of the quarks. This determinant is totally antisymmetric upon exchange of color indices. The rest of the composite wave function must therefore be totally symmetric upon exchange of coordinates. The composite wave function can be rewritten as

$$\Psi = \psi_{\text{color}}\psi_{fc}\psi_{\text{space}} \tag{34}$$

The flavor and angular momentum coupling part can be expressed as

$$\psi_{fc} = (\chi_s[j_1, j_2]1, j_3 JM_z\rangle + \chi_A[j_1, j_2]0, j_3 JM_z\rangle)/\sqrt{2} \tag{35}$$

Here the flavor part consists of only u or d components, the symmetric part upon exchange of the first pair being

$$\chi_s = [duu + udu - 2uud]/\sqrt{6} \tag{36}$$

and the antisymmetric part upon exchange of the first pair being

$$\chi_A = [udu - duu]/\sqrt{2} \tag{37}$$

The combined symmetry of the flavor angular momentum coupling part is maintained by the angular momentum coupling factors having the same symmetry as the corresponding flavor part. J_{12} is 1 for the symmetric flavor part and 0 for the antisymmetric flavor part.

The composite three-body wave function is an eight-component column vector with unknown hyperradial dependence to be determined. The color, flavor, angular momentum coupling, and the orbital factors of the composite wave function are all collected into the factor $U(\Omega)$. The space part of the composite three-quark wave function is determined from the three-body Dirac equation.

5. CORRELATION OF THE FLUX TUBE CONSTANT TO THE NUCLEON REST ENERGY

In the two-component notation, the square of the Dirac Hamiltonian is more nearly diagonal than is the linear Dirac Hamiltonian. After squaring, the only off-diagonal terms involve the commutator of the kinetic energy and the potential terms. The quark potential is taken as a scalar

$$S = b(\beta_1 r_1 + \beta_2 r_2 + \beta_3 r_3) \tag{38}$$

where the subscripts denote the particle label; then the off-diagonal terms of

the squared Hamiltonian are constants. In the overall rest frame of the system, where the total momentum vanishes, using hyperspherical coordinates, one obtains

$$\left[-\nabla^2 + b^2\rho^2 + in\left(\sum_j \beta_j \alpha_{jr}\right)\right]\Psi_0 = W_0\Psi_0 \tag{39}$$

where $W_0 = E_0^2$ and $n = 2b/3$. The zero subscript implies no external electromagnetic interaction is included in the unperturbed system. If the squared Hamiltonian problem were solved exactly, the eigenfunction would be the same as for the linear Hamiltonian problem.

This squared Hamiltonian eigenvalue problem is solved to estimate appropiate values for the flux tube constant b in a massless quark model with no short-ranged one-gluon exchange potential included. In the Dirac two-component notation, the composite three-quark wave function can be written as an eight-component column vector. In the hypercentral approximation that includes only the $(1/2^+)^3$ configuration there exist symmetry relations between these components that reduce the number of unknowns to four [27]. After integrating over the color, flavor, spin, and hyperangular parts of the wave function, the $\nabla^2 R(\rho)U_K(\Omega)$ goes to

$$\langle U_K(\Omega)\nabla^2 R_K(\rho)U_K(\Omega)\rangle = \rho^{-5}\, d/d\rho\, [\rho^5\, dR_K/d\rho] - K(K+4)R_K/\rho^2 \tag{40}$$

Including only the diagonal terms, the eigenfunctions are those of the harmonic oscillator:

$$R_K(\rho) = N_K\, \rho^K \exp(-b\rho^2/2) \tag{41}$$

with the eigenvalue $(2K + 6)b$. This eigenvalue is an energy squared, as b has the units of GeV^2. N_K is the normalization constant for the uncoupled components. K has the value zero for the component that survives in the nonrelativistic limit and increases by one for each lower quark component in the composite three-quark wave function. K ranges from zero to three for the various components of the $(1/2^+)^3$ assumed configuration for the nucleon. The off-diagonal terms are included to first order only, so that the components of the column eigenfunction are coupled, but the shape of each component is unchanged. The components of the wave function pick up an additional coefficient A_K, where now

$$R_K(\rho) = A_K N_K \rho^K \exp(-b\rho^2/2) \tag{42}$$

The coefficients and the eigenvalue W are determined from the determinant

$$\begin{bmatrix} 6b - W & 3n & 0 & 0 \\ n & 8b - W & 2n & 0 \\ 0 & 2n & 10b - W & n \\ 0 & 0 & 3n & 12b - W \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \end{bmatrix} = 0 \tag{43}$$

The normalization condition for the coupling coefficients can be written as

$$1 = A_0^2 + 3A_1^2 + 3A_2^2 + A_3^2 \tag{44}$$

due to the symmetry relations reducing the eight-component equation to a four-component equation. This has the solution

$$W/b = 5.3944 \tag{45}$$

Critchfield [11] exactly solved the one-body massless Dirac equation with a scalar linear potential and found

$$W_{\text{onebody}}/b = 2.6226 \tag{46}$$

Comparing these two results, we see Abe and Fujita's [1] conclusion that the three-body squared energy would be twice the one-body squared energy is quite accurate since 5.39 is almost twice 2.62. If b is chosen to be 0.1632 GeV^2, then $W^{1/2}$, the nucleon rest energy, would be reproduced by this scalar string potential, massless quark model for the proton. In the absence of a one-gluon exchange potential or other mechanism to split the Δ rest energy from the nucleon rest energy, perhaps the flux tube constant should reproduce their average rest energy, 1.087 GeV, in which case $b = 0.2190$ GeV^2. The axial charge of the proton with this wave function is 1.478, compared to the experimental value of 1.26. This shows the approximate solution needs more contribution to the norm from the lower components of the composite three-quark wave function. The rms charge radius of this approximate wave function is 0.5087 fermi, less than the experimental value of 0.83.

6. CONCLUSIONS

Proton electric and magnetic polarizabilities reported here depend on the flux tube constant. A scalar flux tube potential has been used with massless quarks to determine the unperturbed quark wave functions in the nucleon. In the model with massless quarks, a scalar flux tube confining potential, and no one-gluon exchange potential, the nucleon rest energy is proportional to $b^{1/2}$ and the electric polarizability is approximately proportional to $1/b$.

The polarizabilities are calculated in second-order perturbation theory. Three values for the flux tube constant are considered. One matches the

Table I. Nucleon Polarizabilities Versus the Flux Tube Constant

b (GeV^2)	α_P (10^{-4} f^3)	β_P (10^{-4} f^3)	$\alpha_P - \alpha_N$ (10^{-4} f^3)	Comment
0.1632	18.75	2.373	6.25	Proton energy fit (1)
0.18	16.14	2.031	5.38	Meson spectra Studies
0.219	12.02	1.513	4.01	Δ nucleon mass average

proton rest energy, one matches meson spectra [7], and one matches the Δ proton average rest energy. The neutron/proton electric polarizability ratio is 2/3 in this three-valence-quark model of the nucleon. The neutron/proton magnetic polarizabilities are also in the ratio of 2/3. This is in contrast to chiral perturbation theories, where these ratios are 1:1. Sum rule analyses [8, 14] suggest that electric polarizability of the proton minus that for the neutron has the value of $3.8 \times 10^{-4} f^3$. Experiments for the proton electric polarizability [15, 18] report $(12.1 \pm 0.8 \pm 0.5) \times 10^{-4} f^3$ and for the proton magnetic polarizability $(2.1 \mp 0.8 \mp 0.5) \times 10^{-4} f^3$, where the reversal of signs indicates that Baldwin's [2] sum rule was used as a constraint. The experimental values for the neutron electric polarizability are difficult, but reported values are [24] $(12.0 \pm 1.5 \pm 2.0) \times 10^{-4} f^3$ from neutron scattering off heavy nuclei, and more recently [17] $(0 \pm 5) \times 10^{-4} f^3$.

The polarizations as a function of flux tube constant are given in Table I. The values for the flux tube constant are similiar to those used in various constituent quark analyses [9, 12, 13, 18, 20]. Best agreement with experiment is for the flux tube constant to match the average of the Δ nucleon rest energies. The neutron electric polarizability is predicted to be 8.01×10^{-4} f^3 for a flux tube constant of 0.219 GeV^2. The neutron electric polarizability experiments are difficult, but more precise values are needed. Such experiments would sharply discriminate between the chiral perturbation and the current quark mass model predictions.

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